Canonical Transform Methods for Analysis of Radio Occultations

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Summary. We discuss the application of canonical transform (CT) methods for analyzing radio occultations with multipath behavior. In the present work we discuss a mapping to the representation of approximate impact parameter. This method generalizes the full spectrum inversion (FSI) method for the case of non-circular orbits. The method is based on mapping the field by a Fourier integral operator that maps directly from the measured time-dependent field to the impact parameter representation without first doing a back propagation. Furthermore, our method allows for simple, asymptotic direct modelling of wave propagation. We perform processing of simulated radio occultations and show that our method provides the same accuracy and resolution as the standard CT method.

Keywords: Canonical transform, Fourier integral operators, multipath

1 Introduction

The processing of radio occultation data involves the reconstruction of the geometric optical rays from measurements of wave fields, possibly with multipath behavior. The standard retrieval methods use the connection between ray direction and Doppler frequency and can therefore only be applied in single-ray areas. Transforming radio occultation measurements from the space to the impact parameter representation was recently shown to be a powerful retrieval method for signals with multipath behavior [1, 2] (for a discussion of backpropagation and radio-holographic methods see, e.g., Ref. [1]). The transformation is given by a Fourier integral operator (FIO) whose phase function is the generating function of the geometric optical canonical transform (CT). If each value of the impact parameter, p, occurs not more than once, this transform allows for disentangling multiple rays. The CT method provides high accuracy and resolution (about 50 m) of refraction angle profiles.

Another inversion method capable of handling multipath behavior is the full-spectrum inversion (FSI) method [3]. The FSI method also transforms the complex field to the impact parameter representation. This is accomplished by a Fourier transform of the complete record of the measured, complex field multiplied by a reference signal.

A synthesis of the CT and FSI inversion methods was discussed in Ref. [4]. It is possible to derive a general formula for the FIO applied directly to radio occultation data measured along a generic low-Earth orbiter (LEO) trajectory without the back-propagation (BP) procedure. This FIO is referred to as being of type 2 (FIO2). Along these lines one can develop a CT algorithm of type 2 (denoted CT2) that in analogy to the FSI algorithm effectively amounts to a Fourier transform of the measured field [5]. Furthermore, FIOs can also be used for efficient direct modelling of wave propagation [6].

The rest of this paper is organized as follows: In Sec. 2 we introduce Fourier integral operators of type 2, in Sec. 3 we present the CT2 algorithm, and in Sec. 4 we present simulation results. In Sec. 5 we summarize and conclude.

2 Fourier integral operators of type 2

We consider the complex field $u(y) = A(y) \exp(ik\Psi(y))$ recorded along the observation trajectory parameterized with some generic coordinate y, which can be e.g. time, arc length, or satellite-to-satellite angle θ [3]. The generic FIO2 operator that maps the field from the (y, η) to the (p, ξ) representation (η and ξ are the canonical momenta associated to y and p) reads [5]:

$$\hat{\Phi}_2 u(p) = \sqrt{\frac{-ik}{2\pi}} \int a_2(p, y) \exp(ikS_2(p, y)) u(y) dy \,. \tag{1}$$

The functions a_2 and S_2 are denoted amplitude and phase functions. The phase space coordinates are linked by the following canonical transform:

$$\eta = -\partial S_2(p, y)/\partial y, \qquad \xi = \partial S_2(p, y)/\partial p. \tag{2}$$

The transformed field is expressed as $w(p) \equiv \hat{\Phi}_2 u(p) \equiv A'(p) \exp(ik\Psi'(p))$, where $A'(p)$ and $\Psi'(p)$ can be found using the stationary phase method.

The differential equations (2) define the phase function from the known canonical transform $(y, \eta) \to (p, \xi)$. The new momentum ξ is a known function of impact parameter and bending angle. In particlar, it can be chosen to be equal to bending angle ϵ [5, 7]. The bending angle ϵ can thus be found from the momentum ξ and impact parameter p. For a circular occultation geometry the FIO2 reduces to the Fourier transform.

Thus, for the reconstruction of the refraction angle profile from the measurements of the wave field along the orbit we apply the FIO2 operator (1), which produces a function $\hat{\Phi}_2 u(p) = A'(p) \exp(ik\Psi'(p))$ of impact parameter p. The derivative of its eikonal $\Psi'(p)$ with a negative sign is equal to the refraction angle $\epsilon = \epsilon(p, y_s(p))$, where $y_s(p)$ is the coordinate of the trajectory point, where the ray with impact parameter p was observed [$y_s(p)$ is the stationary point of the oscillating integral (1)].

The amplitude function $a_2(p, y)$ of the FIO does not play any role in the computation of refraction angles. For example, we could set $a_2(p, y) \equiv 1$. However, the correct definition of the amplitude of the transformed wave field is necessary for the retrieval of atmospheric absorption. For the derivation of the amplitude function $a_2(p, y)$, we use energy conservation. For a generic coordinate y, we must introduce a measure $\mu(p, y)$ and replace dy with μdy. The expression for the amplitude function reads [5]:

$$a_2(p, y) = \sqrt{\mu \left| \frac{\partial^2 S_2(p, y)}{\partial p \, \partial y} \right|}, \tag{3}$$

with the measure derived from energy conservation:

$$\mu(p, y) = \left(\sqrt{r_L^2 - p^2} \sqrt{r_G^2 - p^2} \right)^{1/2} \times \left| \dot{\theta} - \frac{\dot{r}_G}{r_G} \frac{p}{\sqrt{r_G^2 - p^2}} - \frac{\dot{r}_L}{r_L} \frac{p}{\sqrt{r_L^2 - p^2}} \right|^{1/2}. \tag{4}$$

3 Canonical transform method of type 2: approximate impact parameter representation

The FIO2 defined by the phase and amplitude functions S_2 and a_2 solves the problem of the extraction of refraction angles from measurements of the complex field along a satellite trajectory, directly without back propagation. However, this operator cannot be implemented as a Fourier transform and therefore its numerical implementation, especially for high frequencies such as 10-30 GHz, is slow. Here we shall describe an approximation that allows for writing the FIO2 operator in the form of a Fourier transform [5].

Consider the measured complex field $u(t) = A(t) \exp(ik\Psi(t))$ and corresponding momentum $\sigma = d\Psi/dt$. We use an FIO2 associated with the canonical transform from the (t, σ) to the (p, ξ)-representation. The impact parameter p is a known function of t, σ: $p = p(t, \sigma)$. Instead of exact impact parameter we introduce its approximation $\tilde{p}$ (with momentum $\tilde{\xi}$):

$$\tilde{p}(t, \sigma) = p_0(t) + \frac{\partial p_0}{\partial \sigma} (\sigma - \sigma_0(t)) = f(t) + \frac{\partial p_0}{\partial \sigma} \sigma, \tag{5}$$

$$f(t) = p_0(t) - \frac{\partial p_0}{\partial \sigma} \sigma_0(t), \tag{6}$$

where $\sigma_0(t)$ is a smooth model of Doppler frequency, $p_0(t) = p(\sigma_0(t), t)$, and $\partial p_0 / \partial \sigma = \partial p / \partial \sigma|_{\sigma = \sigma_0(t)}$. We compute $\sigma_0(t)$ by differentiation of the eikonal with a strong smoothing over approximately 2 s time interval. We parameterize the trajectory with the coordinate $Y = Y(t)$, where we use the notation Y in order to distinguish between this specific choice of the trajectory coordinate and the generic coordinate y. For brevity we use the notation $u(Y)$ instead of $u(t(Y))$. For the coordinate Y and the corresponding momentum η we use the following definitions:

$$dY = \left(\frac{\partial p_0}{\partial \sigma}\right)^{-1} dt = \frac{\partial \sigma}{\partial p_0} dt, \qquad \eta = \frac{\partial p_0}{\partial \sigma}\sigma. \tag{7}$$

Then, we can write the linear canonical transform: $\tilde{p} = f(Y) + \eta, \tilde{\xi} = -Y$, where $f(Y)$ is short notation for $f(t(Y))$. The generating function of this canonical transform follows from the differential equation: $dS_2 = \tilde{\xi}\, d\tilde{p} - \eta\, dY = -Y\, d\tilde{p} - (\tilde{p} - f(Y))\, dY$, with the result: $S_2(\tilde{p}, Y) = -\tilde{p}Y + \int_0^Y f(Y')\, dY'$.

Because $\left|\partial^2 S_2 / \partial\tilde{p}\partial Y\right| = 1$, the amplitude function (3) equals $\sqrt{\mu}$ and it can be approximated by: $a_2(\tilde{p}, Y) = (\sqrt{r_L^2 - \tilde{p}^2}\sqrt{r_G^2 - \tilde{p}^2})^{1/2}$ [5]. In addition, the amplitude function $a_2(\tilde{p}, Y)$ can be replaced with $a_2(\tilde{p}, Y_s(\tilde{p}))$, where $Y_s(\tilde{p})$ is the stationary point, and factored out from within the integral. The resulting operator can be written as the composition of adding a model, $ik \int f(Y) dY$, to the phase, the Fourier transform, and an amplitude factor:

$$\hat{\Phi}_2 u(\tilde{p}) = \sqrt{\frac{-ik}{2\pi}} a_2(\tilde{p}, Y_s(\tilde{p})) \int \exp\left(-ik\tilde{p}Y\right) \exp\left(ik \int_0^Y f(Y')\, dY'\right) u(Y)\, dY. \tag{8}$$

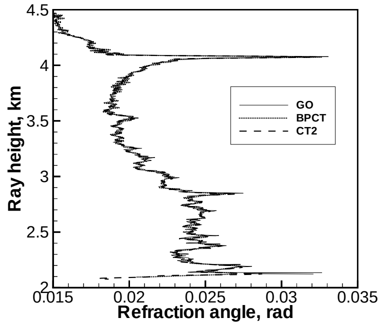
The function $Y_s(\tilde{p})$ equals $-\tilde{\xi}$, where the momentum $\tilde{\xi}$ is the derivative of the eikonal of the integral term in (8). This operator maps the wave field to the representation of the approximate impact parameter $\tilde{p}$. Practically, the difference between p and $\tilde{p}$ is small and can be neglected.

The FIO2 mapping (8) generalizes the FSI method [3]: FSI uses a composition of a phase model with a Fourier transform with respect to the angle θ. Our definition of the coordinate Y takes into account the generic occultation geometry (and for circular orbits $Y = \theta$). We also use a more accurate derivation of the amplitude function a_2. We refer to this CT inversion technique based on the FIO of the second type as the CT2 algorithm.

4 Numerical simulations

Next, we compare the performance of the CT2 inversion technique introduced above and the standard composition of BP and CT techniques. We model a spherically symmetric atmosphere using a high-resolution tropical radio sonde profile of refractivity. We simulate radio occultation signals using multiple phase screens (MPS) and asymptotic, direct modeling (A) (see [5, 6]) for the standard GPS frequencies.

The simulated radio occultation signals were sampled at realistic rates and processed by the CT and CT2 inversion algorithms and the results of the reconstruction of refraction angle profile were compared with the exact geometric optical solution. The comparison presented in Fig. 1 indicates a very



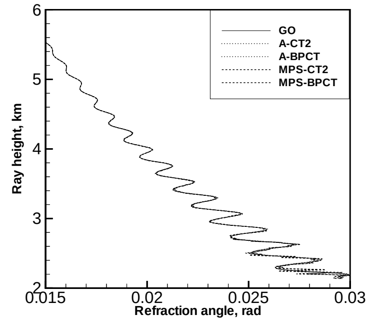


Fig. 1. (Left) Comparison of different modifications of the CT technique. Refraction angle profiles as functions of ray height (impact parameter minus Earth's curvature radius): (1) reference geometric optical solution (GO, solid line), (2) standard composition of BP and CT (BPCT, dotted line), and (3) CT2 algorithm (dashed line).

Fig. 2. (Right) Refraction angle profiles from geometric optical ray tracing and from simulated radio occultation data: (1) reference GO profile (GO, solid line), (2) asymptotic simulation processed by CT2 (A-CT2, dotted line), (3) asymptotic simulation processed by BP+CT (A-BPCT, dotted line), (4) MPS simulation processed by CT2 (MPS-CT2, dashed line), and (5) MPS simulation processed by BP+CT (MPS-BPCT, dashed line).

good agreement between both algorithms and the geometric optical solution. All the differences are in small scales below 50 m, which cannot be effectively resolved for the GPS frequencies, due to diffraction inside the atmosphere.

We also performed simulations with a simple spherically-symmetrical phantom $n(z) = 1 + N_0 \exp\left(-\frac{z}{H}\right)\left[1 + \alpha \cos\left(\frac{2\pi z}{h}\right) \exp\left(-\frac{z^2}{L^2}\right)\right]$, where z is the height above the Earth's surface, $N_0 = 300 \times 10^{-6}$ is the characteristic refractivity at the Earth's surface (300 N-units), $H = 7.5$ km is the characteristic vertical scale of refractivity field, $\alpha = 0.003$ is the relative magnitude of the perturbation, $h = 0.3$ km is the period of the perturbation, $L = 3.0$ km is the characteristic height of the perturbation area.

Figure 2 shows the geometric optical refraction angle profile and the results of the inversion of the simulated data. We present four combinations of the two simulation techniques: (1) the FIO asymptotic solution (A) and (2) multiple phase screens (MPS); and the two inversion techniques: (1) CT2 and (2) the standard combination of BP and CT. The figure shows good agreement between the GO solution and the retrieved refractivity profile. The strongest deviations of retrieved refraction angles from the reference GO profile are observed for processing MPS simulations in the lowest 200 m. This can be accounted for by the diffraction on the Earth's surface.

5 Conclusions

Different techniques for processing radio occultations based on Fourier integral operators have a common point: they utilize an FIO that maps the measured wave field to the impact parameter representation. The original CT method was based on first applying a back propagation of the measured signal. In contrast to this, the FSI and CT2 methods are applied directly to the measured field. The advantage of such methods is that they do not assume stationarity of the GPS satellite. Furthermore, these methods allow for efficient numerical implementations based on FFTs by mapping to an approximate impact parameter representation.

Another application of FIOs is the direct modeling [5, 6]. The asymptotic solution of the direct problem uses the mapping of the geometric optical solution in the impact parameter representation to the standard coordinate representation. A method based on the inverse CT2 is very efficient numerically because it can be implemented as the composition of the geometric optical solution and a single Fourier transform. This is important for direct modeling and for processing radio occultation data at high frequencies (10-30 GHz), where the computation of diffractive integrals may be numerically slow.

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