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Geodesy calculations in ROPP

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Document Signature Table

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1 Background

The ROPP software requires functions to convert the vertical coordinate from geopotential height (Z) to geometric height (h).

The geometric height is defined as an altitude scale with respect to a reference ellipsoid which takes account for the shape of the Earth. It is assumed that geometric heights in the ROPP software are expressed relative to the WGS-84 (World Geodetic System 1984) ellipsoid.

The geopotential height relates altitude to graviational geopotentials and is a function on geometric altitude (h) and geodetic latitude (ϕ) as

$$Z(h,\phi) = \frac{\Phi(h,\phi)}{g_{WMO}} = \frac{1}{g_{WMO}} \int_0^h g(h,\phi) dh \tag{1.1}$$

where g_{WMO} is a reference gravity, defined by WMO as the normal gravity at a latitude of 45° with a constant value of 9.80665 ms^{-2}. The geopotential Φ describes the potential energy per unit mass at a given location. Dividing by g_{WMO} gives a height scale above the reference ellipsoid.

The geometric and geopotential height scales are related as

$$Z(h,\phi) = \frac{g(h,\phi)}{g_{WMO}} \frac{R(\phi)h}{R(\phi)+h} \tag{1.2}$$

where $R(\phi)$ is the radius of Earth at latitude ϕ and $g(h,\phi)$ is the normal gravity at point (h,ϕ).

This document describes the functions used in ROPP to compute $g(h,\phi)$, $R(\phi)$ and to convert between geometric and geopotential height scales. The functions distributed with ROPP v1.0 (see `ropp_tools/geodesy/`) use a method based on the Smithsonian Meteorological Tables (SMT). Further details are provided by Mahoney (2001) who lists expressions based on SMT 1968 (List 1968). There are some slight differences between the implementation of these routines in ROPP v1.0 (based on SMT 1985) and those listed by Mahoney (2001), perhaps due to changes between different editions of SMT (Marquardt, code comment). In any case, Mahoney (2001) suggests that it is inappropriate to derive parameters based on SMT as these are based on International Ellipsoid 1935 which preceeds and differs from WGS-84.

Mahoney (2001) provided alternative expressions to derive geodetic parameters with reference to the WGS-84 ellipsoid. These are listed in this document for clarity together with the SMT relationships. Functions to compute geodetic parameters relative to the WGS-84 ellipsoid are to be included in future versions of the ROPP software (`ropp_tools/geodesy_som`).

1.1 Gravity

The Smithsonian Meteorological Tables give an expression for the normal gravity at latitude ϕ, which accounts for its dependency on h. Mahoney (2001) lists this as

$$g_{SMT}(\phi) = g_{WMO}(1 - 2.637236 \times 10^{-3} \cos(2\phi) - 5.821355 \times 10^{-6} \cos^2(2\phi) \tag{1.3}$$

The version implemented in ROPP v1.0 (`ropp_tools/geodesy/gravity_smt.f90`) computes

$$g_{SMT}(\phi) = 9.780456(1 + 5.2885 \times 10^{-3}\sin^2(\phi) - 5.9 \times 10^{-6}\sin^2(2\phi) \quad (1.4)$$

Mahoney (2001) used Somigliana's equation to derive the normal gravity

$$g_s(\phi) = g_e(\frac{1 + k_s sin^2\phi}{\sqrt{1 - e^2 sin^2\phi}}) \quad (1.5)$$

where g_e is the gravity at the equator (9.7803253359 ms^{-2}), and k_s is Somigliana's constant related to the Earth shape and gravity at the equator and pole (1.931853$\times 10^{-3}$) and e is the eccentricity (0.081819). This is implemented in `ropp_tools/geodesy_som/gravity_som.f90`.

Figure 1.1 shows that the differences between $g(\phi)$ values calculated using Somigliana's equation (Equation 1.5) and the SMT versions (Equations 1.3 and 1.4) are negligible.

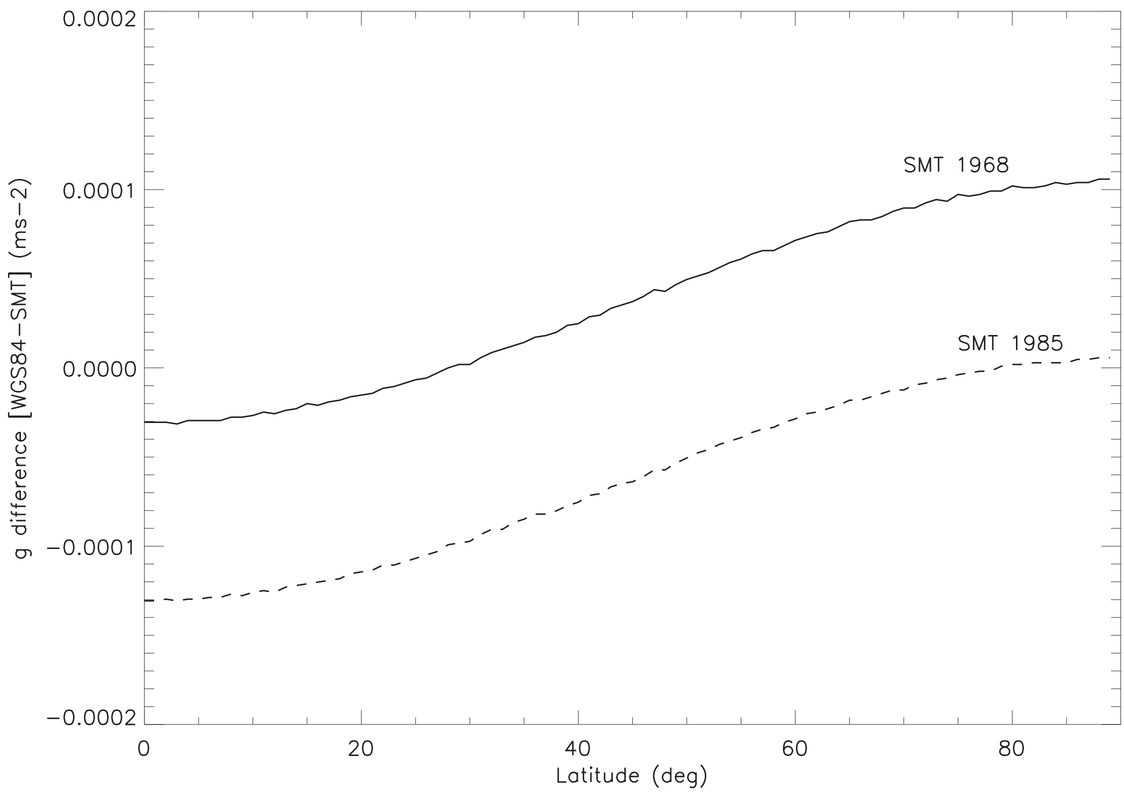


Figure 1.1: Difference between gravity computed using Somigliana's equation (Equation 1.5) for WGS-84 ellipsoid and the values derived using the Smithsonian Meteorological Tables (Equations 1.3 and 1.4)

1.2 Earth radius

The Smithsonian Meteorological Tables give an expression for the effective radius of Earth at latitude ϕ as

$$R_{SMT}(\phi) = \frac{2g_{SMT}(\phi)}{-\frac{d}{dz}g_{SMT}(\phi)} \quad (1.6)$$

Mahoney (2001) lists an expression for the denominator in Equation 1.6 so that

$$R_{SMT}(\phi) = \frac{2g_{SMT}(\phi)}{3.085462 \times 10^{-6} + 2.27 \times 10^{-9} * \cos(2\phi) - 2 \times 10^{12}\cos(4\phi)} \quad (1.7)$$

This equation is implemented in ROPP v1.0 (in `ropp_tools/geodesy/r_eff_smt.f90`)

From Somigliana's equation, Mahoney (2001) shows that

$$R_s(\phi) = \frac{a}{1 + f + m - 2f\sin^2\phi} \quad (1.8)$$

where a is the semi-major axis (6378.1370 km), f is the flattening (0.003352811) and m is the gravity ratio (0.003449787). This is implemented in `ropp_tools/geodesy_som/r_eff_som.f90`.

Figure 1.2 shows the difference between Earth radius values computed using Equations 1.7 and 1.8. Referencing values relative to the WGS-84 ellipsoid rather than assuming that expressions derived from the SMT can lead to differences in R of up to 350 m ($R_s < R_{SMT}$ at 90°). Even larger differences occur if the value of $g(\phi)$ calculated using Equation 1.4 is used.

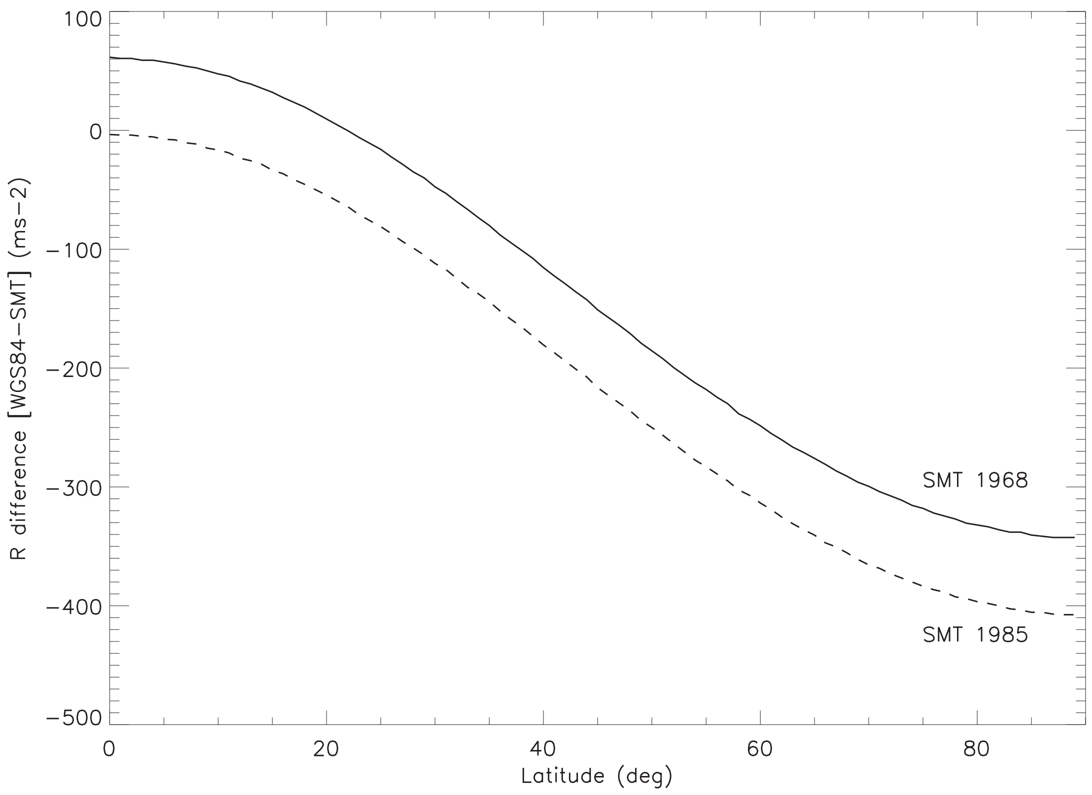


Figure 1.2: Difference between Earth radius computed using Somigliana's equation (Equation 1.8) for WGS-84 ellipsoid and the values derived using the Smithsonian Meteorological Tables (Equation 1.7) using the g values described in Equations 1.3 and 1.4)

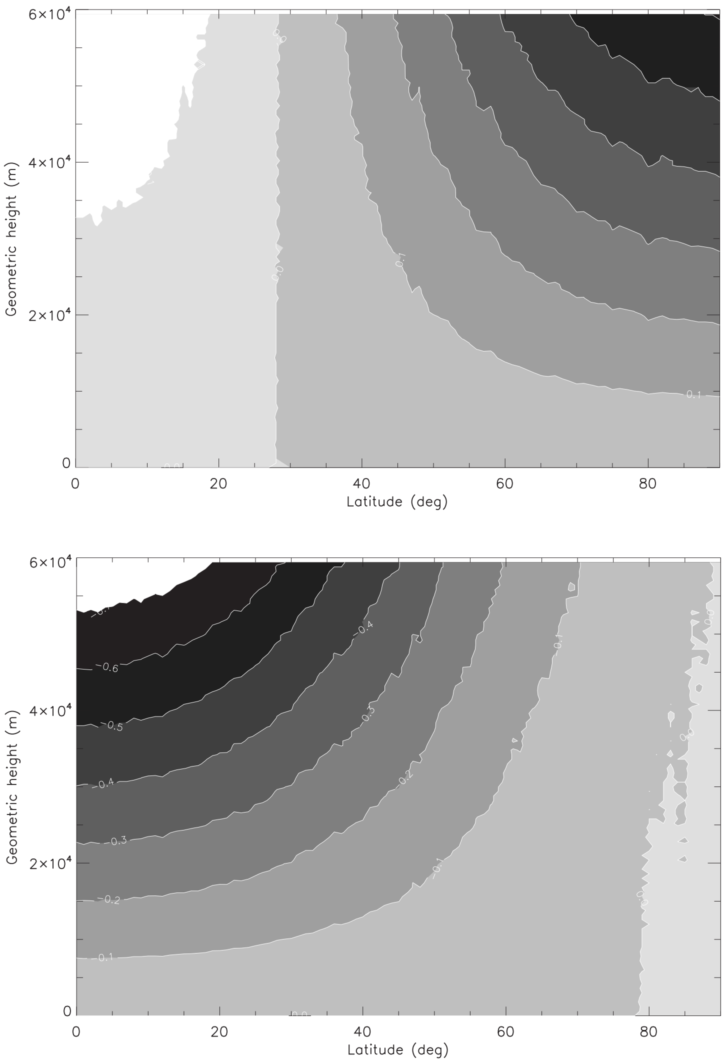
1.3 Geopotential height

Routines to convert between geopotential (Φ) and geometric height (h) are provided in ROPP v1.0 `ropp_tools` module by the functions `geodesy/geopotential2h_smt.f90` ($h = f(\Phi)$)

and `geodesy/h2geopotential_smt.f90` ($\Phi = f(h)$). Similar routines to convert between geopotential height (Z) and geometric height (h) using the Somigliana implementation are provided in future ROPP versions by `geodesy_som/geopotential2geometric_som.f90` and `geodesy_som/geometric2geopotential_som.f90`.

The impact of the differences in $g(\phi)$ and $R(\phi)$ resulting from the different reference ellipsoids on the conversion from geometric to geopotential altitudes (Equation 1.2) is illustrated in Figure 1.3. This shows that differences of up to 0.8 m in the computed altitude scale may occur when converting between geometric and geopotential heights of about 60 km. This corresponds to an error of 0.001$\%$!

In summary, errors in the geopotential and geometric height conversions based on the assumptions of the Smithsonian Meteorological Tables are only a few tens of metres. Errors in the effective Earth radius calculation may be several hundred metres however. For clarity and consistency, it is therefore recommended that the conversion routines based on Somigliana's equation are implemented in all ROPP GPSRO applications.



(a)

(b)

Figure 1.3: Difference between geopotential height computed using values of $g(\phi)$ and $R(\phi)$ based on Somigliana's equation for WGS-84 ellipsoid and equations based on the Smithsonian Meteorological Tables using the expressions corresponding to (a) SMT 1968 and (b) SMT 1985. Contours show the difference $Z_s(h,\phi) - Z_{SMT}(h,\phi)$ for each value of h and ϕ in m.

Bibliography

[1] R. J. List, Smithsonian Meteorological Tables, Smithsonian Institution Press, Washington, 1968.

[2] M. J. Mahoney, A discussion of various measures of altitudes, URL: http://mtp.jpl.nasa.gov/notes/altitude/altitude.html, 2001.

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