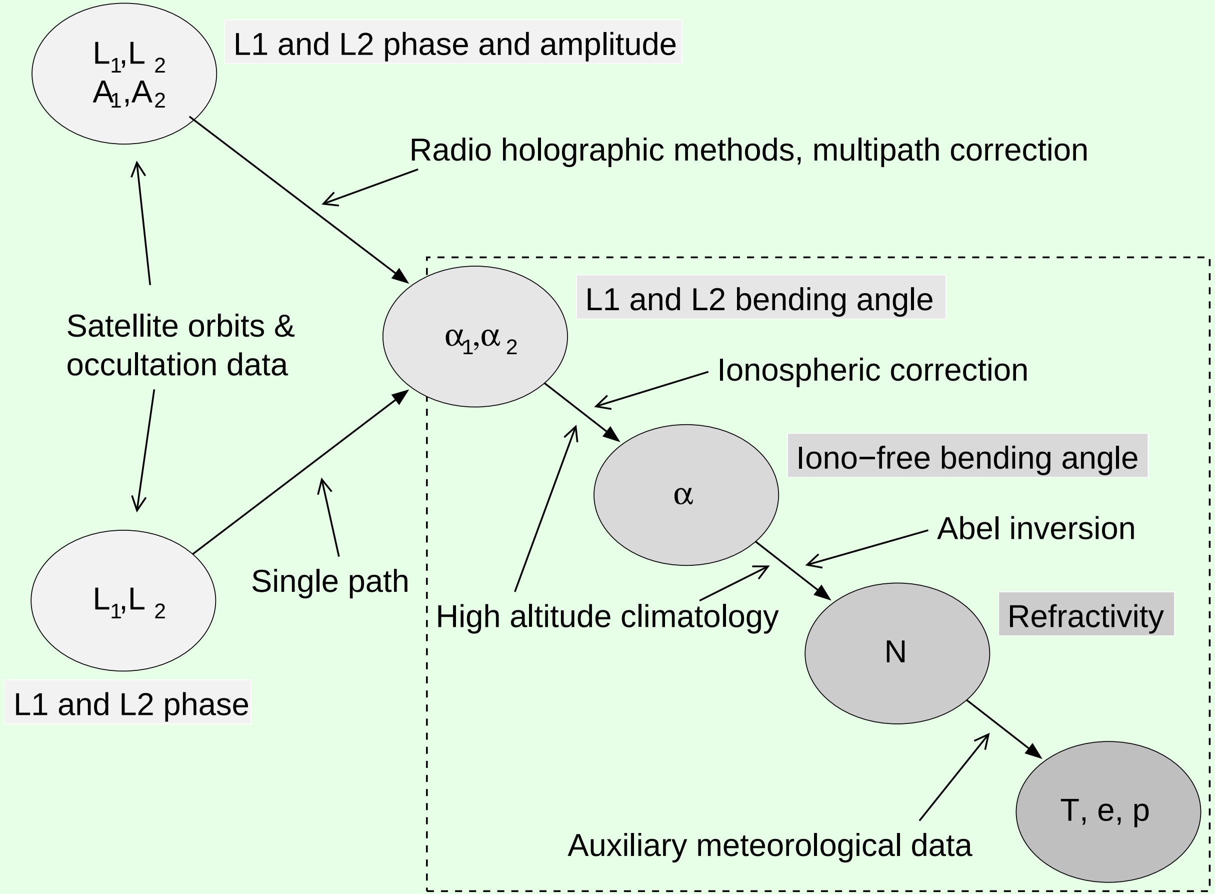
Generic Processing of GPS RO and Microwave Occultations

Stig Syndergaard
Danish Meteorological Institute, Copenhagen, Denmark

Joint OPAC-4 & GRAS SAF Climate & IROWG-1 Workshop, Graz, Austria, September 6–11, 2010

GPS RO measurements & processing



Bending angle → refractivity

$$\alpha(a)$$

$$\downarrow$$

Abel integral transform (e.g., Fjeldbo 1971)

$$\alpha(a) = -2a \int_{r_0}^{\infty} \frac{\mathrm{d}\ln n/\mathrm{d}r}{\sqrt{n^2r^2 - a^2}}\mathrm{d}r \quad \Longleftrightarrow \quad n(r_0) = \exp\left(\frac{1}{\pi}\int_a^{\infty} \frac{\alpha(x)}{\sqrt{x^2 - a^2}}\mathrm{d}x\right)$$

$$r_0 = \frac{a}{n(r_0)} \quad , \quad N(r_0) = 10^6 \times (n(r_0) - 1)$$

$$\downarrow$$

$$N(r)$$

- The Abel integral transform relies on the assumption of spherical symmetry
- It provides a simple and unique solution to an otherwise under-determined inverse problem
- A few issues: 1) ionospheric correction; 2) statistical optimization – how to handle the upper boundary

Ionospheric correction of bending angles

The ionosphere-free bending angle is formed from the derived bending angles at the same impact parameter (Vorob'ev and Krasil'nikova 1994):

$$\alpha(a) = \frac{f_1^2 \alpha_1(a) - f_2^2 \alpha_2(a)}{f_1^2 - f_2^2}$$

- In practice one has to consider filtering of small-scale ionospheric residuals and excessive L2 noise (Sokolovskiy, 2009)
- The GRAS SAF uses a so-called optimal linear combination aproach (Gorbunov, 2002), combining ionospheric correction and statistical optimization in a Bayesian framework

Statistical optimization

- Formally, we need bending angles to infinite altitudes in order to derive the refractivity (of course we don't have that)
- Bending angles are contaminated with thermal noise and residual noise from ionospheric turbulence
- Fractionally the noise increases exponentially with altitude rendering the bending angle useless at some altitude and above

Optimal estimation of bending angle:

$$\tilde{\alpha}(a) = \alpha_{\rm bg}(a) + \frac{\sigma_{\rm bg}^2}{\sigma_{\rm bg}^2 + \sigma_{\rm obs}^2}[\alpha(a) - \alpha_{\rm bg}(a)]$$

$\alpha_{\rm bg}$ is estimated from a climatological model or other
$\sigma_{\rm obs}$ may be evaluated from the data above the stratosphere
$\sigma_{\rm bg}$ may be fixed (e.g., 20%) or estimated

The GRAS SAF approach

- Based on a spectral representation of the MSIS climatological model transformed to bending angle space
- Global search in model space + scaling and offset (Gobiet and Kirchengast, 2004; Lohmann, 2005)
- Least squares fit of $A\alpha_{\mathrm{gmsis}}^{B}$ to the observed (non-optimized) bending angle between 40 and 60 km, and then...

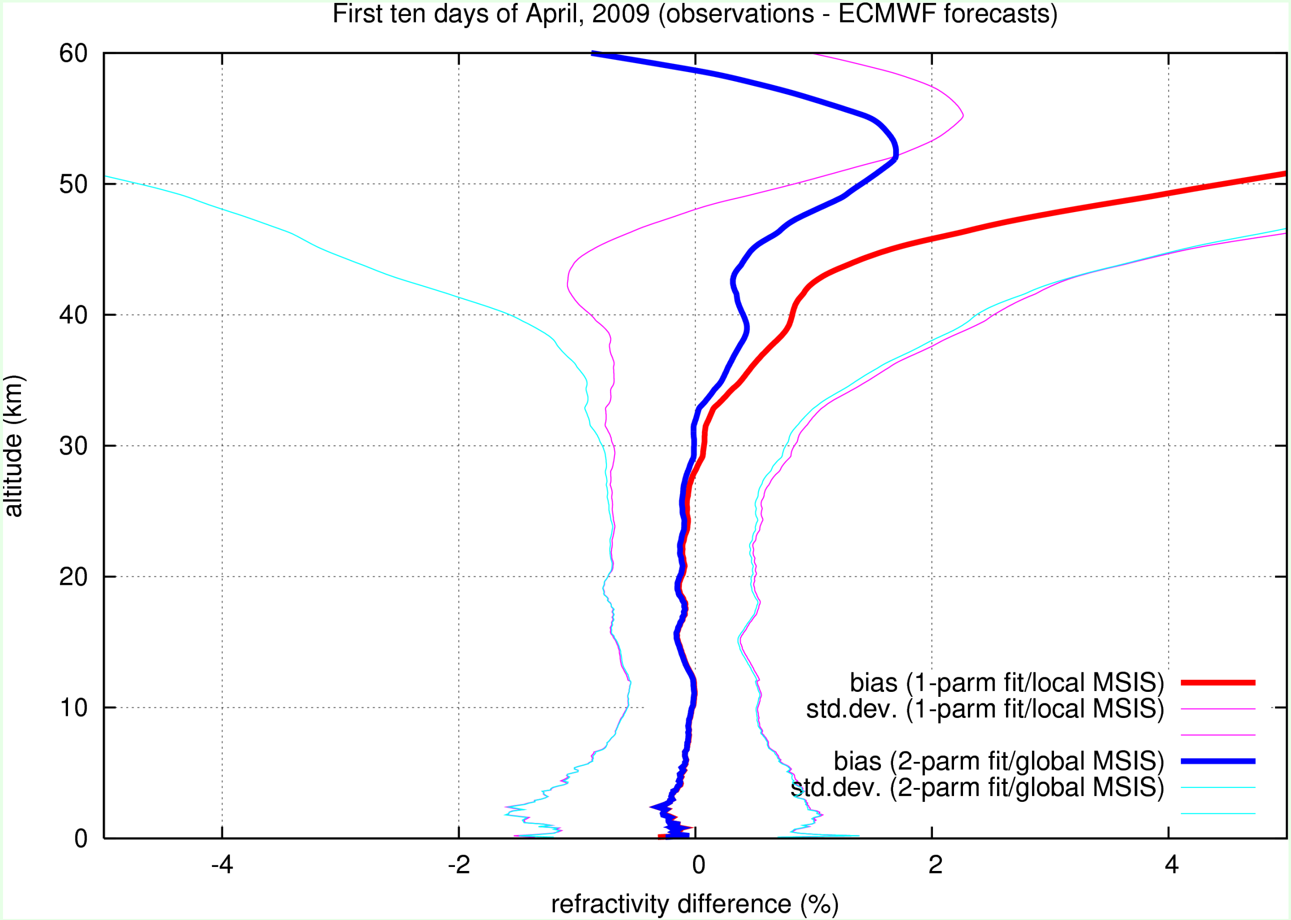
$$\min[(\ln A)^2 + (B-1)^2]_{\mathrm{global\,search}} \quad \Rightarrow \quad \alpha_{\mathrm{bg}} = A\alpha_{\mathrm{gmsis}}^{B}$$

Optimal linear combination:

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \mathbf{K} \begin{pmatrix} \tilde{\alpha} \\ \Delta\alpha_{\mathrm{I}} \end{pmatrix} \quad , \quad \mathbf{K} = \begin{pmatrix} 1 & f_2^2/(f_1^2 - f_2^2) \\ 1 & f_1^2/(f_1^2 - f_2^2) \end{pmatrix}$$

$$\begin{pmatrix} \tilde{\alpha} \\ \Delta\alpha_{\mathrm{I}} \end{pmatrix} = \begin{pmatrix} \alpha_{\mathrm{bg}} \\ \langle\Delta\alpha_{\mathrm{I}}\rangle \end{pmatrix} + \mathbf{K}^{\dagger} \begin{pmatrix} \alpha_1 - \alpha_{\mathrm{bg}} - \langle\Delta\alpha_{\mathrm{I}}\rangle f_2^2/(f_1^2 - f_2^2) \\ \alpha_2 - \alpha_{\mathrm{bg}} - \langle\Delta\alpha_{\mathrm{I}}\rangle f_1^2/(f_1^2 - f_2^2) \end{pmatrix}$$

Refractivity statistics against ECMWF



Refractivity → pressure & temperature

Refractivity equation:

$$N \approx 77.6\frac{p}{T} + 3.73 \times 10^5 \frac{e}{T^2}$$

- Two terms: a dry (or hydrostatic) term and a wet term
- The wet term can be neglected at temperatures less than ∼240 K (i.e., at few kilometers above the surface at high latitudes and above ∼10 km at tropical latitudes)

Neglecting the wet term:

$$N(r) \longrightarrow \boxed{\begin{array}{ll} N = 77.6\dfrac{p}{T} \quad , & p = \rho R_{\mathrm{d}} T \\ \dfrac{\mathrm{d}p}{\mathrm{d}z} = -\rho g \quad , & z = r - r_{\mathrm{curv}} \end{array}} \longrightarrow p(z), T(z)$$

Deriving temperature and water vapor

1. Include additional information about the actual temperature profile and solve directly for water vapor (iterative procedure)
2. One-dimensional variational technique optimally combining the refractivity profile with information from an NWP model
3. The 'COSMIC approach'

Method number 1:

$$N(z), T_{\mathrm{apriori}}(z)$$

$$\downarrow$$

$$e = T^2 \frac{N - 77.6(p_\mathrm{d} + e)/T}{3.73 \times 10^5} \quad , \quad \frac{\mathrm{d}(p_\mathrm{d} + e)}{\mathrm{d}z} = -(\rho_\mathrm{d} + \rho_\mathrm{w})g$$

$$p_\mathrm{d} = \rho_\mathrm{d} R_\mathrm{d} T \quad , \quad e = \rho_\mathrm{w} R_\mathrm{w} T$$

$$\downarrow$$

$$e(z), p_\mathrm{d}(z)$$

An optimal solution toward T, p, and e

Method number 2 (variational retrieval):

- Include information about errors in a priori temperature, pressure and water vapor, as well as errors in the observed refractivity

$$N(z), T_{\mathrm{apriori}}(z), p_{\mathrm{apriori}}(z), e_{\mathrm{apriori}}(z)$$

$$\downarrow$$

Minimizing the following cost function:

$$J(\mathbf{x}) = (\mathbf{x} - \mathbf{x}_\mathrm{b})^\mathrm{T}\mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}_\mathrm{b}) + (\mathbf{N}_\mathrm{obs} - \mathbf{N}(\mathbf{x}))^\mathrm{T}\mathbf{R}^{-1}(\mathbf{N}_\mathrm{obs} - \mathbf{N}(\mathbf{x}))$$

$\mathbf{x}$ is the state vector to be solved for

$\mathbf{x}_\mathrm{b}$ is the a priori state vector

$\mathbf{N}(\mathbf{x})$ is the refractivity equation

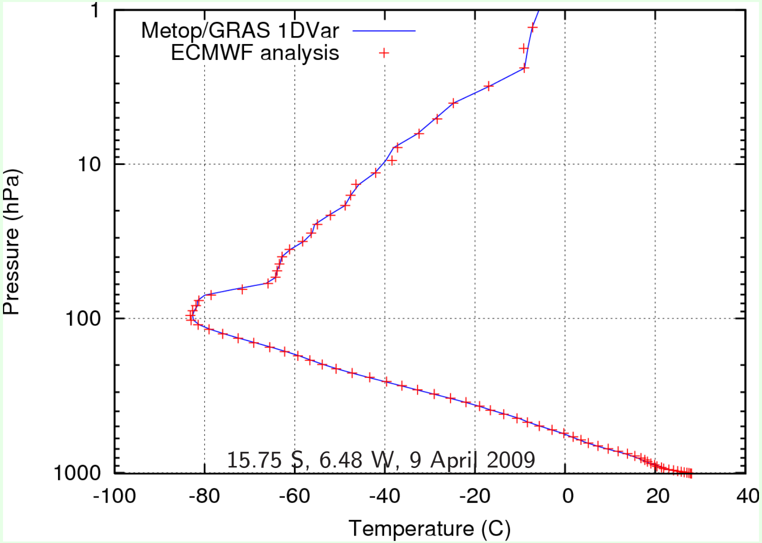
$\mathbf{B}$ is the a priori error covariance matrix

$\mathbf{R}$ is the observation + representativeness error covariance matrix

$$\downarrow$$

$$T(z), p(z), e(z)$$

The GRAS SAF approach



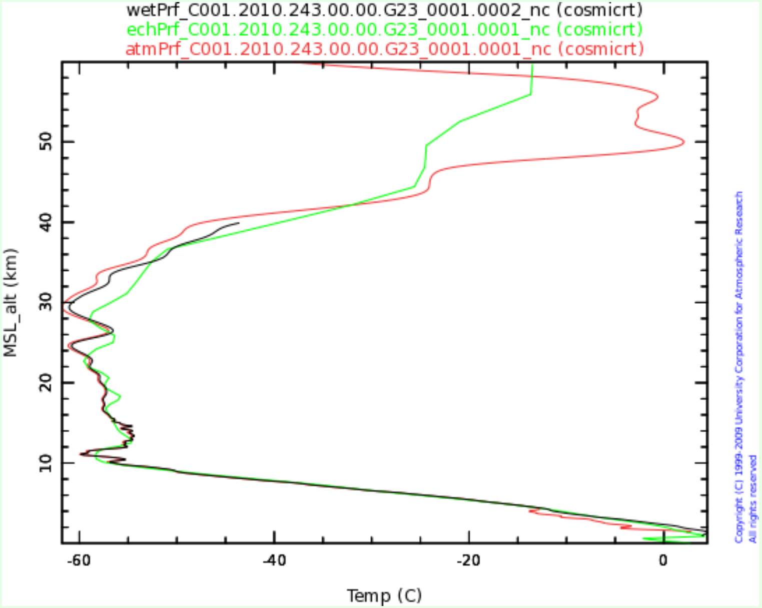
- Observations and forecast weighted according to their error covariance
- An optimal mixture between model and observation
- One disadvantage: retrieved temperature and water vapor inconsistent with observed refractivity

The COSMIC approach

Method number 3 (COSMIC approach):

- Gives much more weight to the observations
- Seeks to minimize the influence from NWP fields, but still separate out temperature and water vapor

- Solution (almost) consistent with observed refractivity
- Retrieved temperature (almost) in-line with dry temperature where water vapor is insignificant
- Observed small-scale structure is preserved in the solution



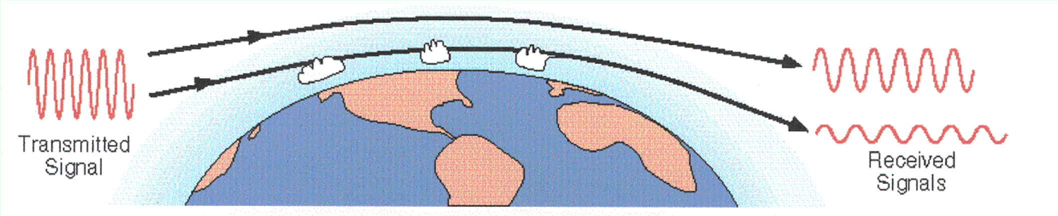
A similar alternative – taking the full step

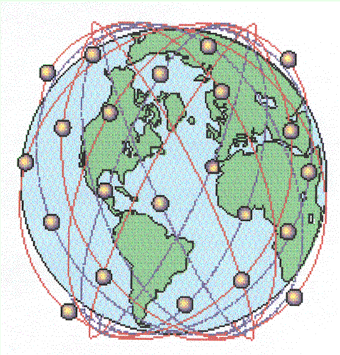
- Give full weight to the observations (zero error; $\mathbf{S}_{\mathrm{O}} = 0$)
- Solution fully consistent with observed refractivity ($N_{\mathrm{sol}} = N_{\mathrm{obs}}$)
- Retrieved temperature fully in-line with dry temperature where water vapor is insignificant ($T_{\mathrm{sol}} = T_{\mathrm{dry}}$ where $e = 0$)
- Information content from observation fully preserved
- NWP model add only information on the relative contributions of dry and wet terms, based on its representation of these variables ($\mathbf{x}_{\mathrm{B}}$) and their error co-variances ($\mathbf{S}_{\mathrm{B}}$)
- NWP *analyses* (where the specific RO profile presumably has already been assimilated) can be used as $\mathbf{x}_{\mathrm{B}}$ without including the information from the RO data twice (since the RO in itself does not contain information about the relative contributions of dry and wet terms)

Standard 1Dvar: $\mathbf{x} = \mathbf{x}_{\mathrm{B}} + \mathbf{S}_{\mathrm{B}}\mathbf{K}^{\mathrm{T}}(\mathbf{K}\mathbf{S}_{\mathrm{B}}\mathbf{K}^{\mathrm{T}} + \mathbf{S}_{\mathrm{O}})^{-1}(\mathbf{y} - \mathbf{K}\mathbf{x}_{\mathrm{B}})$ [notation after Rodgers (2000)]

Alternative: $\mathbf{x} = \mathbf{x}_{\mathrm{B}} + \mathbf{S}_{\mathrm{B}}\mathbf{K}^{\mathrm{T}}(\mathbf{K}\mathbf{S}_{\mathrm{B}}\mathbf{K}^{\mathrm{T}})^{-1}(\mathbf{y} - \mathbf{K}\mathbf{x}_{\mathrm{B}}) = \mathbf{x}_{\mathrm{A}} + \mathbf{S}_{\mathrm{A}}\mathbf{K}^{\mathrm{T}}(\mathbf{K}\mathbf{S}_{\mathrm{A}}\mathbf{K}^{\mathrm{T}})^{-1}(\mathbf{y} - \mathbf{K}\mathbf{x}_{\mathrm{A}})$

Microwave occultations





Atmospheric limb sounding (occultations) using multi-frequency signals between LEO satellites

Observations:

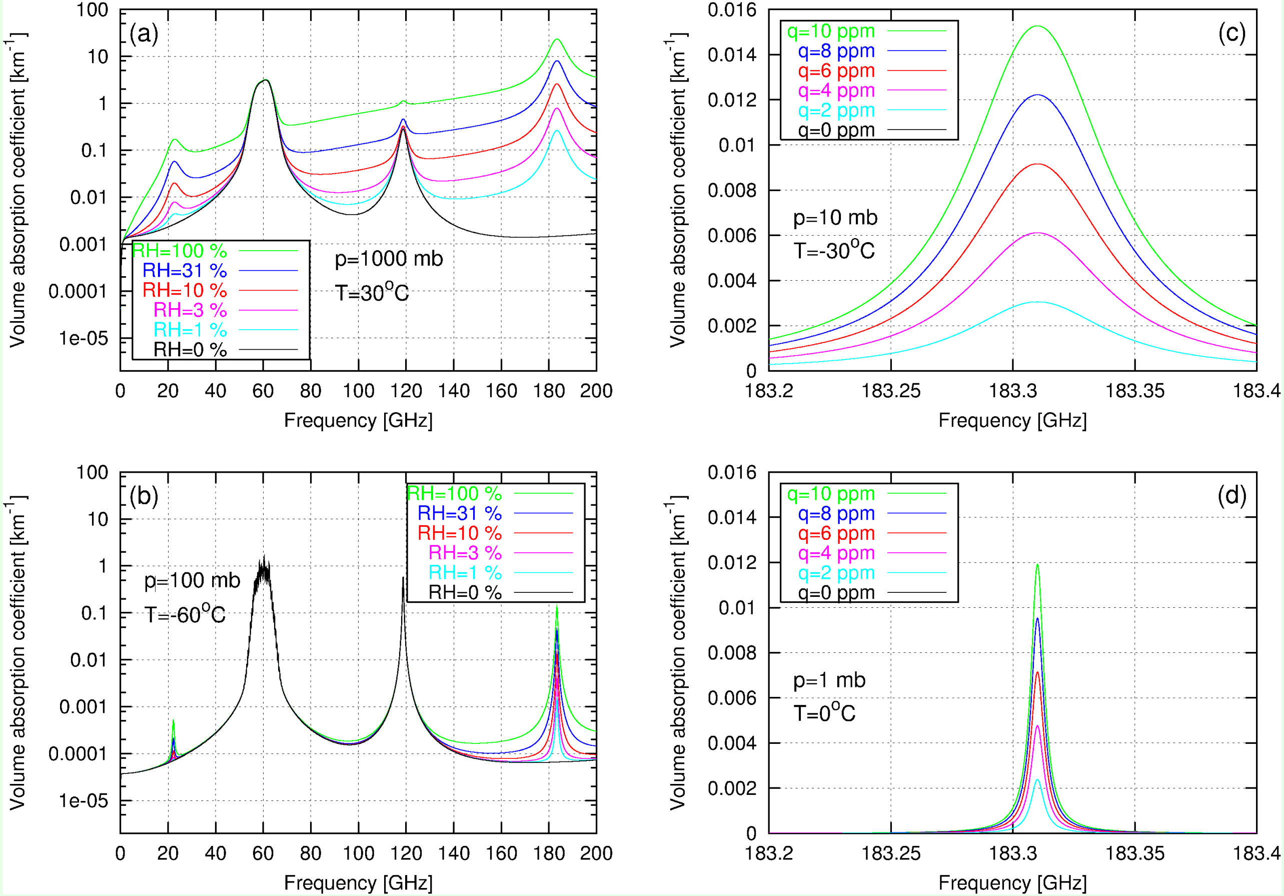
1. Refraction (via phase measurements)
2. Absorption (via amplitude measurements)

Products: Profiles of temperature, pressure, water vapor, ozone, ...

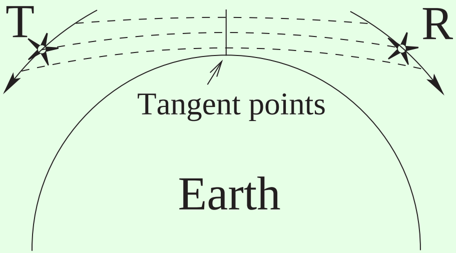
Frequencies:

- 9-32 GHz and 178-183 GHz for moisture sounding
- 184-196 GHz for ozone sounding

Absorption spectra below 200 GHz



Basic principles of the observations



Optical path: $L = \int n \mathrm{d}s$

Optical depth: $\tau = \int k \mathrm{d}s$

n is the real part of the refractive index

k is the volume absorption coefficient (related to the imaginary part of the refractive index)

Normalized intensity: $I/I_0 = \zeta \exp(-\tau)$

The difference in optical depth, $\Delta\tau$, between two signals (at two different frequencies) is obtained by signal intensity ratioing

Basic principles of the retrieval

- Inversion to obtain n and Δk

$$L(t) \;\rightarrow\; \alpha(a) \;\rightarrow\; \boxed{\begin{array}{c} n(r) = \exp\left(\frac{1}{\pi}\int_a^\infty \frac{\alpha(x)}{\sqrt{x^2-a^2}}\mathrm{d}x\right) \\ \Delta k(r) = -\frac{1}{\pi}\frac{\mathrm{d}a}{\mathrm{d}r}\int_a^\infty \frac{\mathrm{d}\Delta\tau/\mathrm{d}x}{\sqrt{x^2-a^2}}\mathrm{d}x \\ a = rn(r) \end{array}} \;\rightarrow\; n(r)$$

$$I(t) \;\rightarrow\; \Delta\tau(a) \;\rightarrow\; \cdots \;\rightarrow\; \Delta k(r)$$

- Solving a set of non-linear equations to obtain profiles of $p, \; e, \; T$

$$\Delta k = F_1(p,e,T) \qquad n = F_2(p,e,T) \qquad \mathrm{d}p/\mathrm{d}z = F_3(p,e,T)$$

Handling the integration to infinity

- Extending the bending angle profile; e.g., log-linear extrapolation to infinity
- Integration to some high altitude where the bending angle can be neglected (above 100 km)
- Analytical (approximate) expression assuming log-linear bending angle above a_{top}:

$$\int_a^\infty \frac{\alpha(x)\mathrm{d}x}{\sqrt{x^2-a^2}} \approx \int_a^{a_{\text{top}}} \frac{\alpha(x)\mathrm{d}x}{\sqrt{x^2-a^2}} + \frac{\alpha(a_{\text{top}})\sqrt{\pi H}}{\sqrt{a_{\text{top}}+a}} \exp\left(\frac{a_{\text{top}}-a}{H}\right)\mathrm{erfc}\left(\sqrt{\frac{a_{\text{top}}-a}{H}}\right)$$

- An old idea of mine: Use background/climatological bending angle defined to arbitrary high altitude (spectral representation) and apply substitution $(x=\sqrt{a^2+(c\ln y)^2})$ to effectively integrate to infinity:

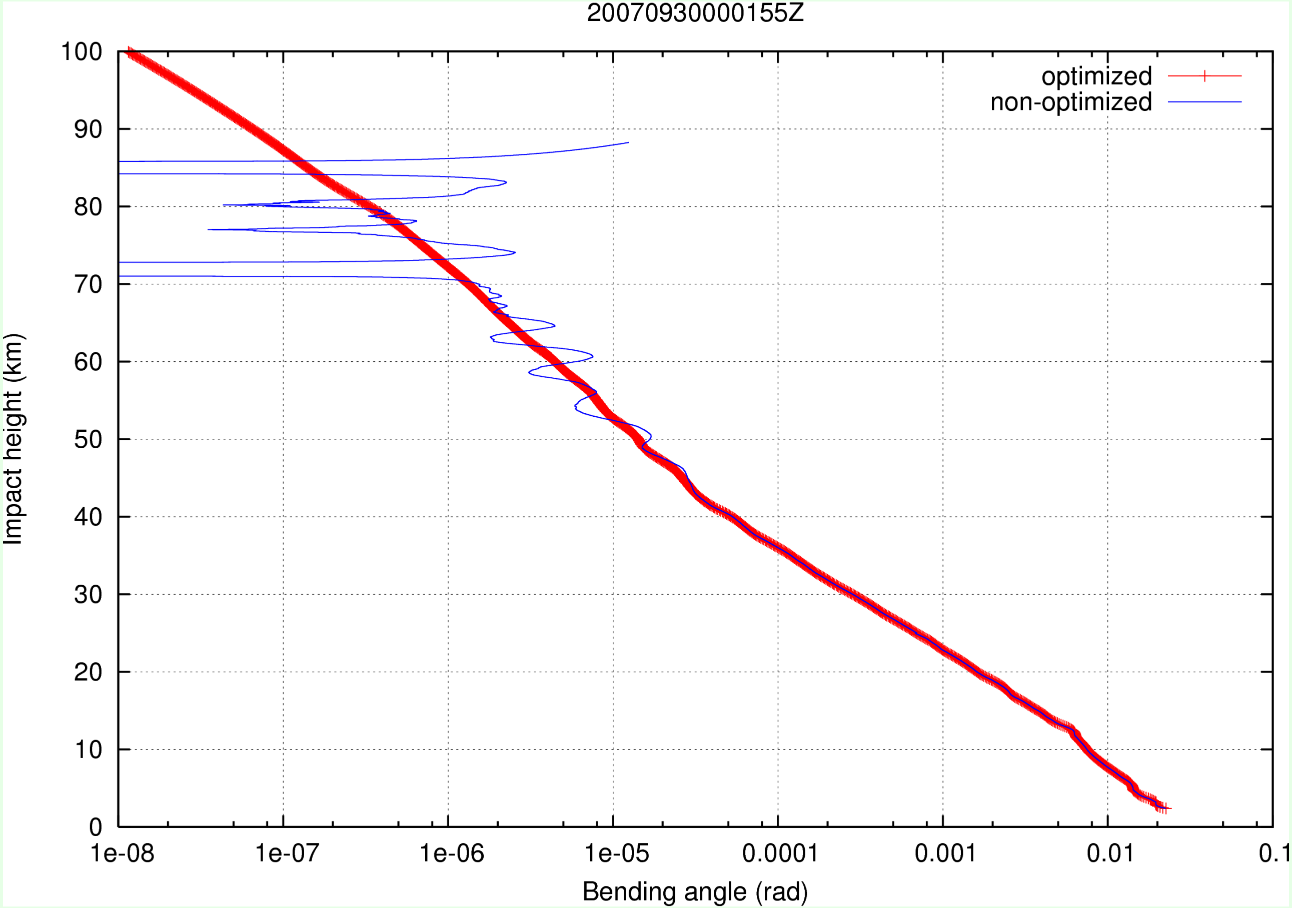
$$\int_a^\infty \frac{\alpha(x)\mathrm{d}x}{\sqrt{x^2-a^2}} = c\int_0^1 \frac{\alpha(y)}{xy}\mathrm{d}y$$

... and the lower limit

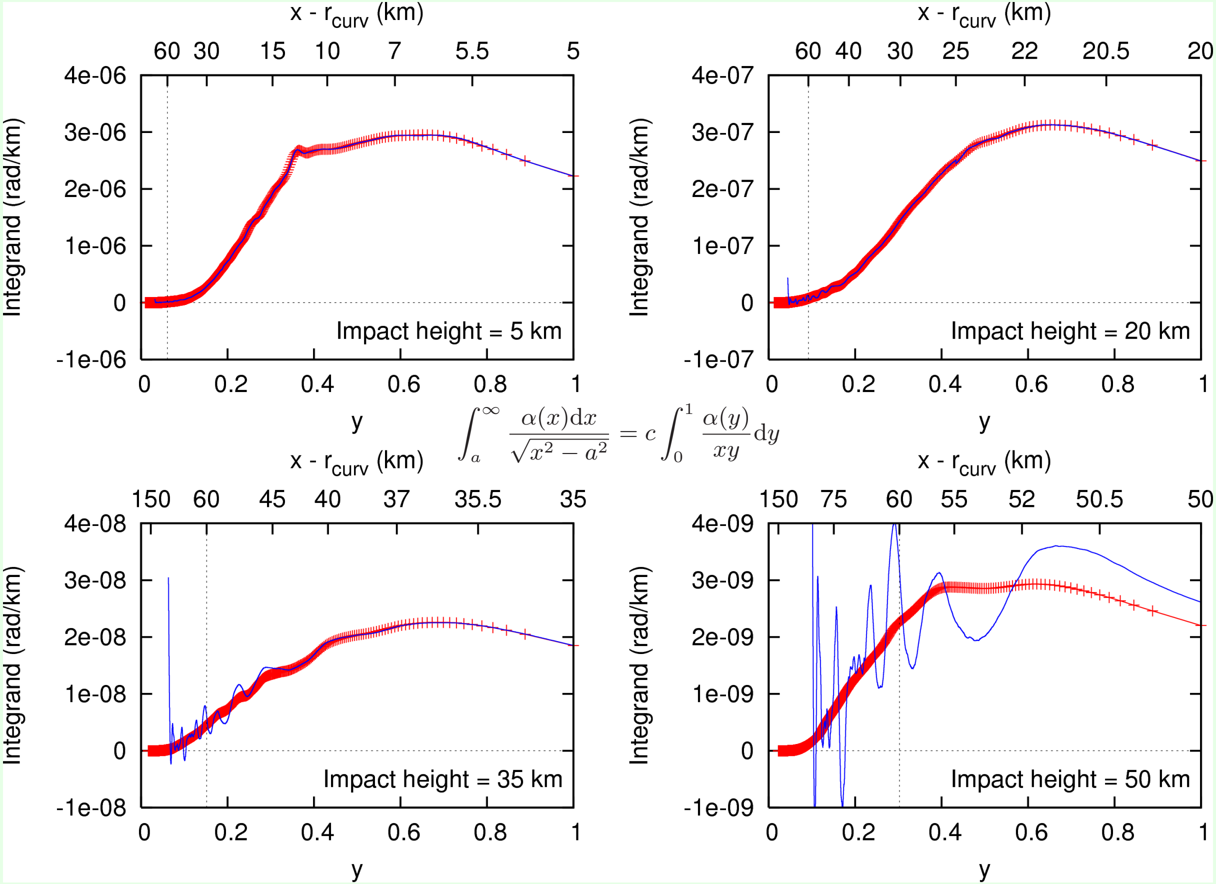
We rarely talk about it, but we all do something:

- $x = \sqrt{a^2 + (c \ln y)^2} \quad \Rightarrow \quad \int_a^\infty \frac{\alpha(x)\mathrm{d}x}{\sqrt{x^2 - a^2}} = c \int_0^1 \frac{\alpha(y)}{xy}\mathrm{d}y \quad , \qquad \frac{\alpha(y)}{xy}\Big|_{y\to 0} \to 0$
- $\alpha(x) = Ax + B \quad \Rightarrow \quad \int_a^b \frac{\alpha(x)\mathrm{d}x}{\sqrt{x^2 - a^2}} = A\sqrt{b^2 - a^2} + B \ln(\frac{b + \sqrt{b^2 - a^2}}{a})$
- integration by parts : $\int_a^\infty \frac{\alpha(x)\mathrm{d}x}{\sqrt{x^2 - a^2}} = -\int_a^\infty \ln\left(\frac{x}{a} + \sqrt{\left(\frac{x}{a}\right)^2 - 1}\right)\frac{\mathrm{d}\alpha}{\mathrm{d}x}\mathrm{d}x$
- $x = \sqrt{s^2 + a^2} \quad \Rightarrow \quad \int_a^\infty \frac{\alpha(x)\mathrm{d}x}{\sqrt{x^2 - a^2}} = \int_0^\infty \frac{\alpha(s)\mathrm{d}s}{\sqrt{s^2 + a^2}}$
- $x = a\cosh\theta \quad \Rightarrow \quad \int_a^\infty \frac{\alpha(x)\mathrm{d}x}{\sqrt{x^2 - a^2}} = \int_0^\infty \alpha(\theta)\mathrm{d}\theta$
- ... and the one you use...

Example from GRAS data



A closer look . . .



$$\int_a^\infty \frac{\alpha(x)\,\mathrm{d}x}{\sqrt{x^2 - a^2}} = c \int_0^1 \frac{\alpha(y)}{xy}\mathrm{d}y$$